DETAILED SYLLABUS - M.Sc. MATHEMATICS

Paper 1: ALGEBRA

Unit 1:	Conjugacy relation– Class equation for finite groups and its applications – Sylow's theorems (for theorem 2.12.1, first proof only) Chapter 2: section 2.11 and 2.12 (omit Lemma 2.12.5)
Unit 2:	Solvable groups – Direct products – Finite abelian groups – Modules Chapter 5: Section 5.7 (Lemma 5.7.1,5.7.2, Theorem 5.7.1) Chapter 2: Section 2.13 and 2.14 (Theorem 2.14.1 only) Chapter 4: Section 4.5
Unit 3:	Linear transformations: Canonical forms – triangular forms – Nilpotent transformations Chapter 6: Section 6.4 & 6.5
Unit 4:	Jordan form – rational canonical form Chapter 6: Section 6.6 & 6.7
Unit 5:	Trace and Transpose- Hermitian, unitary, Normal Transformation, real quadratic form Chapter 6: Section 6.8, 6.10 & 6.11(Omit 6.9)

Books for Reference and study:

Content and treatment as in **I. N. HERSTEIN, Topics in Algebra** (II Edition) Wiley Eastern Limited, New Delhi,1975.

Books for supplementary reading and reference:

1. M. Artin, Algebra, Prentice Hall of India,1991. P. B. Bhattacharya, S. K. Jain & S. R.
2. Nagpaul, Basic Abstract Algebra (II Edition), Cambridge University press, 1997. (Indian Edition).
3. I. S. Luthar and I. B. S. Passi, Algebra, Vol I-Groups (1996); Vol II Rings, Narosa Publishing House, New Delhi,1999.
4. D. S. Malik, J. N. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, McGraw Hill (International Edition), New York,1997.
5. N. Jacobson, Basic Algebra, Vol I & II W. H. Freeman (1980); also published by Hindustan Publishing Company, New Delhi.

PAPER 2: REAL ANALYSIS -1

Unit 1: Element of point set topology - Euclidean space R - Open balls & open sets in R - Closed sets - Adherent point & Accumulation points - Closed sets & adherent points - Bolzano-Weierstrass theorem - The cantor intersection theorem - The Lindelof covering theorem - The Heine-Borel covering theorem - Compactness in R -Functions of Bounded Variation - Monotonic functions - Functions of Bounded variation - Total variation - Additive property of Total Variation - Total variation considered as a function Functions of bounded variation expressed as the difference of increasing functions - Continuous function of bounded variation

Unit 2: The Riemann-Stieltjes integral - Notation & Definition - Linear Properties - Integration by parts - Change of variable in a Riemann-Stieltjes Integral - Reduction to Riemann- integral - Step function as a integrators - Upper & Lower integrals - Additive and Linearity Properties of upper and lower integrals - Riemann's condition - Comparsion theorems

Unit 3: Integrator of Bounded variation - Sufficient condition for existence of Riemann-Stieltjes integrals - Necessary conditions for Existence of Riemann-Stieltjes integrals - Mean-value theorems for Riemann-Stieltjes integrals - The integral as a function of the integral -Second Fundamental theorem of integral calculus - Change of variable in a Riemann integral and Second mean theorem for Riemann integrals - Interchanging the order of integration.

Unit 4: Infinite series & Infinite Products - Limit Superior & Limit Inferior of real-Valued sequences - Inserting & removing parentheses - Alternate series - Absolute & Conditional convergence - Dirichlet's test & Abel's test - Rearrangement of series - Riemann theorem on conditionally convergent series - Double sequence - Double series - Rearrangement theorem for Double series - A sufficient condition for equality of iterated series - Multiplication of series - Cesaro Summation - Infinite products.

Unit 5: Sequences of Functions - Pointwise converges of sequences of functions - Examples of sequences of Real-Valued functions - Definition of uniform convergence - Uniform convergence & continuity - The Cauchy condition for uniform convergence of infinite series of functions - Uniform convergence of infinite series of functions - Uniform convergence of Riemann-Stieltjes integration - Uniform convergence and Differentiation - Sufficient conditions for uniform convergence of a series - Uniform convergence and Double sequences.

Books for Reference and study:

Content and treatment as in **Mathematical Analysis (II edition) by Tom. M. Apostol**, Narosa Publishing House

Unit 1: Ch.3, Sec.3.2, 3.3, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 3.11, 3.12 - Ch.6, Sec. 6.2, 6.3, 6.4, 6.5, 6.6, 6.7, 6.8.

Unit 2: Ch.7, Sec. 7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.11, 7.12, 7.13, 7.14

Unit 3: Ch.7, Sec. 7.15, 7.16, 7.17, 7.18, 7.19, 7.20, 7.21, 7.22, 7.25

Unit 4: Ch.8, Sec. 8.3, 8.6, 8.7, 8.8, 8.15, 8.17, 8.19, 8.20, 8.21, 8.22, 8.23, 8.24, 8.25, 8.26

Unit 5: Ch.9, Sec. 9.1, 9.2, 9.3, 9.4, 9.5, 9.6, 9.8, 9.10, 9.11, 9.12

PAPER 3: ORDINARY DIFFERENTIAL EQUATIONS

Unit 1: Linear equations with constant coefficients – Second order homogeneous equations – initial value problems – Linear dependence and independence – Wronskian and a formula for Wronskian – Non homogeneous equation of order two.

Chapter 2: Sections 1 to 6.

Unit 2: Linear equations with constant coefficients – Homogeneous and non homogeneous equation of order n – initial value problems – annihilator method to solve non homogeneous equation – Algebra of constant coefficient operators.

Chapter 2: Sections 7 to 12.

Unit 3: Linear equations with variable coefficients – initial value problems – existence and uniqueness theorems – solutions - to solve a non-homogeneous equation – Wronskian and linear dependence – reduction of the order of a homogeneous equation – homogeneous equations with analytic coefficients – the Legendre equation.

Chapter 3: Sections 1 to 8.

Unit 4: Linear equations with regular singular points – Euler equations – second order equations with regular singular points – Exceptional cases – Bessel function.

Chapter 4: Sections 1 to 4 and 6 to 8. (omit sections 5 and 9).

Unit 5: Existence and Uniqueness of solutions to first order equations – equations with variables separated – exact equation – method of successive approximations – Lipschitz condition – convergence of the successive approximations and the existence theorem.

Chapter 5: Sections 1 to 6 (omit sections 7 to 9).

Books for Reference and study:

1. Content and treatment as in E. A. Coddington – An introduction to Ordinary Differential Equations (3rd printing) – Prentice Hall Ltd., New Delhi 1987.
2. Williams E.Boyce and Richard C.Dl Prima – Elementary Differential equations and boundary value problems – John Wiley and Sons, New York, 1967.
3. George F.Simmons – Differential Equations with applications and historical notes – Tata McGeraw Hill , New Delhi 1974.
4. N.N.Lebedev – Special functions and their applications – Prentice Hall of India, New Delhi 1965.
5. W.T.Reid – Ordinary Differential Equations – Jon Wiley and Sons, New York, 1971.

PAPER 4: DISCRETE MATHEMATICS

Unit 1: The principle of inclusion – exclusion – recurrence relations – generating functions of sequences – coefficients of generating functions – recurrence relations – solving recurrence relations – methods of characteristic roots

Unit 2: Lattices and Boolean Algebra – Lattices as partially ordered sets – Boolean Algebra – Boolean functions.

Unit 3: Graph Theory – Graphs, sub-graphs and trees

Unit 4: Connectivity – Euler tours and Hamilton cycles

Unit 5: Matchings – edge colorings

Books for Reference and study:

Content and treatment as in

1. For Unit 1 - Discrete Mathematics for Computer Scientists and Mathematicians by Joe L. Mott, Abraham Kandel, Theodore P. Baker – Prentice Hall India, New Delhi 1999 – Chapter 2, Section 2.8 and Chapter 3
2. For Unit 2 - Discrete Mathematical Structures with Applications to Computer Science by Trembley and Manohar – Tat McGraw Hill Edition.
   Chapter 4 – Sections 4.1 to 4.3 (Omit sections 4.4, 4.5 and 4.6)
3. For Units 3, 4 and 5 – J. A. Bondy and Murthy, Graph Theory and Applications – Macmillan London 1976.
   Unit 3: Chapter 1 – Section 1.1 to 1.7
   Chapter 2 – Section 2.1 to 2.3
   Unit 4: Chapter 3 – Section 3.1 and 3.2
   Chapter 4 – Section 4.1 and 4.2
   Unit 5: Chapter 5 – Section 5.1 and 5.2
   Chapter 6 – Section 6.1 and 6.2

Books for supplementary reading and reference:

1. A. Gill, Applied Algebra for Computer Science, Prentice Hall Inc., New Jersey.
2. J. L. Gersting Mathematical Structure for Computer Science (3rd Edition) Computer Science Press, New York.
3. S. Witala, Discrete Mathematics- A unified Approach, McGraw Hill Book Co.,
4. Rudolf Lidl and Gunter Pilz, Applied Abstract Algebra, Springer – Verlag, New York, 1984.

PAPER 5: ELECTIVE – I (FUZZY SETS AND SYSTEMS)

Unit 1:	Fundamental notations
	Chapter 1: Sections 1 to 8.
Unit 2:	Fuzzy Graphs
	Chapter 2: Sections 10 to 18.
Unit 3:	Fuzzy relations
	Chapter 2: Sections 19 to 29.
Unit 4:	Fuzzy logic
	Chapter 3: Sections 31 to 36, 39 and 40 (Omit 37, 38 and 41).
Unit 5:	The laws of Fuzzy composition
	Chapter 4: Sections 43 to 49.

Books for Reference and study:

Content and treatment as in **A. Kaufman, Introduction to the theory of Fuzzy Subsets,** Vol. I, Academic Press, New York, 1975.

Books for supplementary reading and reference:

1. H. J. Zimmermann, Fuzzy set theory and its applications, Allied Publishers, Chennai 1996.
2. George J. Klir and Bo Yaun, Fuzzy sets and Fuzzy logics – Theory and Appications, Prentice Hall, India, New Delhi, 2001.

PAPER 6: ALGEBRA – II

Unit 1: Extension field – Transcendence of e

Chapter 5: Section 5.1 and 5.2

Unit 2: Roots of polynomials – More about roots.

Chapter 5: Section 5.3 and 5.5

Unit 3: Elements of Galois theory.

Chapter 5: Section 5.6

Unit 4: Finite fields – Wedderburn's theorem on finite division rings.

Chapter 7: Section 7.1 and 7.2(Theorem 7.2.1 only).

Unit 5: Solvability by radicals – A theorem of Frobenius – Integral Quaternions and the Four-Square theorem.

Chapter 5: Section 5.7 (omit Lemma 5.7.1,5.7.2 and Theorem 5.7.1) - Chapter 7: Section 7.3 and 7.4

Books for Reference and study:

Content and treatment as in **I. N. Herstein, Topics in Algebra (II Edition)** Wiley Eastern Limited, New Delhi, 1975.

Books for supplementary reading and reference:

1. M. Artin, Algebra, Prentice Hall of India,1991.
2. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Basic Abstract Algebra (II Edition) Cambridge University Press, 1997(Indian Edition).
3. I. S. Luther and I. B. S. Passai, Algebra, Vol. I – Groups (1996); vol II Rings, Narosa Publishing House, New Delhi,1999.
4. D. S. Malik, J. N. Mordeson and M. K. Sen, Fundamental of Abstract Algebra, McGraw Hill (International Edition), New York. 1997
5. N. Jacobson, Basic Algebra, Vol I & II W. H. Freeman (1980); also published by Hindustan Publishing Company, New Delhi.

PAPER 7: REAL ANALYSIS - II

Unit 1: The Lebesgue integral: Introduction –the integral of a step function-Monotonic sequence of step functions-upper functions & their integrals-Riemann integral functions as examples of upper functions. The class of Lebesgue integrable functions on a general interval- Basis of properties of the Lebesgue integral –Lebesgue integration & sets of measure zero-The Levi monotone convergence theorems –the Lebesgue dominated convergence Theorem-Lebesgue integral on unbounded intervals as limits of integrals on bounded intervals-Improper Riemann integrals.

Unit 2: Measurable Functions-Continuity of functions defined by Lebesgue integral Differentiation under the integral sign-interchanging the of integration-Measurable set on real line. the Lebesgue integral over arbitrary subsets of R –Inner products &norms-the set L(I) as a semi metricspace –a convergence theorem for series of functions in L(I)-The Riesz-Fisher theorem.

Unit 3: Fourier series: Fourier series & Fourier integrals –Introduction-Orthogonal stem of functions-the theorem on best approximation –The Fourier series of a function relative to an orthogonal system-Proporties of the Fourier coefficients –The Riesz-Fisher theorem –the dirichlet integral- An integral representation for the partial sums of a fourier series –Riemann localization theorem –Sufficient conditions for convergence of Fejer's theorem the Weierstrass approximation theorem.

Unit 4: Multivariable Differential Calculus: Introduction- The directional derivative-directional derivative and Continuity-the total derivative – The total derivative expressed in terms of partial derivatives. the matrix of the linear functions-the Jacobian matix- The Chain rule-Matrix form of the chain rule-the mean value theorem for differential functions-A Sufficient conditions for differentiability- A sufficient conditions for equality of functions from R to R .

Unit 5: Power series -Multiplication of power series –the substitution theorem- Reciprocal of a power series – the Taylor's series generated by a function – Bernstein's theorem – Binomial theorem – Abel limit theorem – Tauber's theorem

Books for Reference and study:

Content and treatment as in **Mathematical Analysis (II Edition) by Tom. M. Apostol**, Narosa Publishing House

Chapter 9: Section 9.14 to 9.23

Chapter 10: Section 10.1 to 10.25 (Omit 10.20)

Chapter 11: Section 11.1 to 11.15 Chapter 12: Section 12.1 to 12.14 (Omit 12.6)

PAPER 8 – PARTIAL DIFFERENTIAL EQUATIONS

Unit 1: Mathematical models and classification of second order equation: Classical equations – Vibrating string – Vibrating membrane – waves in elastic medium – conduction of heat in solids – Gravitational potential – second order equations in two independent variables – canonical forms – equations with constant coefficients–general solution. Chapter 3: Sec 3.1 to 3.6 (Omit 3.5) Chapter 4: Sec 4.1 to 4.4

Unit 2: Cauchy problem: The Cauchy problem – Cauchy Kowalewsky theorem – Homogeneous wave equation – Initial Boundary value problem – Non homogeneous boundary conditions – finite string with fixed ends – Non homogeneous wave equation – Riemann method – Goursat problem – spherical wave equation – cylindrical wave equation. Chapter 5: Sec 5.1 to 5.11.

Unit 3: Method of separation of variables: Separation of variables – vibrating string problem – Existence and uniqueness of solution of vibrating string problem – Heat conduction problem – Existence and uniqueness of solution of heat conduction problem – Laplace and beam equations Chapter 7: Sec 7.1 to 7.7

Unit 4: Boundary value problems: Boundary value problems – Maximum and minimum principles – uniqueness and continuity theorem – Dirichlet problem for a circle, a circular annulus, a rectangle – Dirichlet problem involving Poisson equation – Neumann problem for a circle and a rectangle. Chapter 9: Sec 9.1 to 9.9

Unit 5: Greens function: The Delta function – Green's function – Method of Green's function –Dirichlet Problem for the Laplace and Helmholtz operators – Method of images and Eigen functions Higher dimensional problems – Neumann problem. Chapter 11: Sec 11.1 to 11.10

Books for Reference and study:

Content and treatment as in **Tyn Myint-U and Lokenath Debnath, Partial Differential Equations for scientists and Engineers (Third Edition)**; North Hollan, New York, 1987.

Books for supplementary reading and reference:

1. W. E. Willkams, Partial differential equations, Oxford, Clarenden, 1980.
2. I. N. Sneddon, The use of integral transforms, Tata McGraw Hill, New Delhi,1985.
3. M. M. Smirnov, Second order partial differential equations, Leningrad, 1964.
4. Ian Sneddon, Elements of partial differential equations, McGraw Hill. New Delhi, 1983
5. R. Dennemeyer, Introduction to Partial Differential Equations and Boundary value problems, McGraw Hill Book Company, New York ,1968.
6. M. D. Raisinghania, Advanced Differential Equations, S. Chand & Company Ltd., New Delhi 2001.

PAPER 9 – PROGRAMMING WITH C++

Unit 1: Beginning with C++ - tokens – expressions – control structures. (Chapter 2 and 3)

Unit 2: Functions in C++ - classes and objects. (Chapter 4 and 5)

Unit 3: Constructors and Destructors – operator overloading and type conversions. Chapter 6 and 7

Unit 4: Inheritance – pointers – virtual functions – polymorphisms. (Chapter 8 and 9)

Unit 5: Managing console I/O operations – manipulating strings.(Chapter 10 and 15.)

Books for Reference and study:

Content and treatment as in **Object Oriented Programming with C++by E. Balagursamy**, Tata McGraw Hill.

Practical list:

1. Define a class to represent a bank account, include the following numbers: Name, Acc-no, Acc-Type, Balance. Member functions: To assign initial values, deposit amount, withdraw amount after checking the balance. Write a main program to test the program for handling 10 customers.
2. Define a class string. Use overloaded = operator to compare two string.
3. Write a class called employee that contains a name and employee number. Include a member function to get data from the user for insertion into object, and another function to display the data. Write a main() program to create an array of employee information and accept information from the user and finally print the information.
4. Write a program to use a common friend function to exchange the private values of two classes.
5. Write a program to include all possible binary operator overloading using friend function.
6. Write a program to read an array of integer numbers and sort it in descending order. Use read data, put data, and array max as member functions in a class.
7. Write a program to read two values of time and to find the greater of the two overload the '<' operator for comparison.
8. Write a program to accept employee information such as name, number and salary of 3 employees and display the record of the employees chosen by the user using pointers.
9. Every positive integer 'n' can be expressed as a product of primes.
10. To express GCD(a,b)=ax+by for some integers x and y.
11. To find the value of Mobius function mu(n).
12. To find the value of Euler's Phi-function ($\varphi(n)$).
13. To find the highest power of prime in n!
14. To find the solution of the linear congruence $ax \equiv b(\mathrm{mod}\ m)$.
15. To find the solution of system of linear congruence using Chinese Reminder Theorem.

PAPER 10 – ELECTIVE – II (EDE) PRACTICAL MATHEMATICS

Unit 1: **Elements of Set Theory:** Set – subset – union, intersection, product of two sets – counting of number of elements of union of two or three finite sets – Venn diagrams – simple problems

Unit 2: **Number System:** Natural numbers – addition, multiplication, primes, composites coprimes. Factorization – Integers, Division algorithm, Rationals. Reals. Decimal Representation – representation in Binary forms. Powers – Logarithms – Addition and multiplication of Binary numbers – simple problems

Unit 3: **Geometry:** Shape – Area of two dimensional figures – circle – semicircle – sector of a circle – Triangle, Quadrilateral – Polygons – Regular polygons – Triangularization – Shape – volume – surface area of three dimensional figures. Sphere – hemisphere – cylinder – cone – frustum of cone – combination of these figures – simple problems.

Unit 4: **Statistic:** Classification of data – enumeration – tabulation – measure of central tendency – mean, median and mode – Standard deviation – coefficients of variation – simple problems.

Unit 5: **Probability:** Elementary probability – sample space – evens – mutually exclusive events – addition theorem – independent events - multiplication theorem – Baye's theorem – simple problems.

PAPER 11 – COMPLEX ANALYSIS – I

Unit 1: Cauchy's integral formula: The index of a point with respect to a closed curve – The integral formula – Higher derivatives. Local Properties of analytical functions: Removable Singularities – Taylor's theorem- Series and poles – The local mapping – The Maximum Principle.

Chapter 4: Sec 2.1 to 2.3.

Unit 2: The general form of Cauchy theorem – chains and cycles – simple continuity – homology – the general statement of Cauchy's theorem – proof of Cauchy's theorem – locally exact differentials – multiply connected regions - residue theorem – the argument principle.

Chapter 4: Section 4 – 4.1 to 4.7

Chapter 4: Section 5- 5.1 and 5.2

Unit 3: Evaluation of definite integrals and Harmonic functions – evaluation of definite integrals – definition of harmonic function and basic properties – mean value property – Poisson formula.

Chapter 4: Section 5 – 5.3

Chapter 4: Section 6 – 6.1 to 6.3

Unit 4: Harmonic functions and Power series expansions – Schwarz theorem – the reflection principle – Weierstrass theorem – Taylor's series – Laurent series.

Chapter 4: Section 6.4 and 6.5

Chapter 5: Section 1.1 to 1.3.

Unit 5: Partial Fractions and entire functions – Partial fractions – infinite products – canonical products – Gamma function – Jensen's formula – Hadamard's theorem.

Chapter 5: Sections 2.1 to 2.4

Chapter 5: Sections 3.1 and 3.2.

Books for Reference and study:

Content and treatment as in **Lars f.Ahlfors , Complex Analysis, (3rd Edition),** Mc Graw Hill Book Company; New York, 1979.

Books for supplementary reading and reference:

1. H.A. Prestly, Introduction to Complex Analysis, Clarend Press, Oxford, 1990.
2. J.B. Conway, Functions of one Complex Variable, Springer – Verlag, International Student Edition, Naraso Publishing Co.
3. E. Hille, Analytic Function Theory, (2 Vols), Gonm & Co, 1959.
4. M. Heins, Complex Function Theory, Academic Press, New York, 1968.

PAPER 12 – TOPOLOGY

Unit 1: Metric space – topological space

Chapter 2 and 3

Unit 2: Compactness

Chapter 4

Unit 3: Separation – T_1 spaces and Hausdroff spaces – completely regular space and normal space – Urysohn's lemma and the Tietze extension theorem – the Urysohn imbedding theorem.

Chapter 5: Section 26 to 29

Unit 4: The Stone – Cech compactification – connected spaces – components of a space – totally disconnected spaces – locally connected spaces

Chapter 5: Section 30 and

Chapter 6: Full

Unit 5: Approximation – Weierstrass approximation theorem – Stone – Weierstrass theorem – locally compact Hausdroff spaces – extended stone - Weierstrass theorem.

Chapter 7: Full

Books for Reference and study:

Content and treatment as in **G. F. Simmons, Introduction to Topology and Modern Analysis,** McGraw Hill – International Student edition.

Books for supplementary reading and reference:

1. J. Dugundji, Prentice Hall of India, New Delhi, 1975.
2. James R. Munkers, Topology (2nd Edition), Pearson Education Pvt Ltd, Delhi, 2002 (third Indian reprint).
3. J. L. Kelly, General Topology, Van Nostrand, Reinhold Co., New York.
4. L. Steen and J. Seeback – Counter examples in Topology, Holt Rinehard and Winston, New York, 1970.
5. S. Williams, Gneral Topology, Addison Wesley, Mass., 1970.

PAPER 13 – DIFFERENTIAL GEOMETRY

Unit 1: **Space curves:** -Definitions of a Space Curve- Arc Length – tangent- normal and binormal -curvature and torsion – contact between the curves and surfaces – tangent surfaces-involutes and evolutes- Intrinsic Equations – Fundamental Existence Theorem for Space Curves – Helices.

Chapter 1: Section 1 to 9

Unit 2: **Intrinsic properties of a surfaces:** Definition of a surface – curves on a surface – Surface of revolution – helicoids- metric – Direction coefficients- families of curves - isometric correspondence- Intrinsic properties.

Unit 3: **Geodesics:** - Geodesocs – Canonocal geodesic equations – Normal property of geodesics- Existance Theorems – Geodesic parallels – Geodesic curvature – Gauss-Bonnet Theorem – Gaussian curvature – surface of constant curvature.

Chapter 2: Sections 10 to 18.

Unit 4: **Non intrinsic properties of a surface: -** The second fundamental form – Principal curvature- Lines of curvature- Development associated with space curves and with curves on surface- Minimal surfaces- Ruled surfaces.

Chapter 3: Section 1 to 8.

Unit 5: **Fundamental equations of surface theory:** Parallel surfaces – fundamental existence theorems for surfaces.

Chapter 3: Section 9 to 11.

Books for Reference and study:

Content and treatment as in **T. J. Willmore, An Introduction to Differential geometry,** Oxford University Press, (17th Impression) New Delhi 2002 (Indian Print).

Books for supplementary reading and reference:

1. Struik, D.T. Lectures on Classical Differential Geometry, Addison – Wesley, Mass. 1950.
2. Kobayashi. S. and Nomizu. K.Foundations of Differential Geometry, Interscience Publishers, 1963.
3. Wilhelm Klingenbreg: A course in Differential Geometry, Graduate Texts in Mathematics,
4. Springer-verlag 1978.
5. J.A. Thorape, Elementary topics in Differential Geometry, Under-graduate texts in Mathematics, Springer- Verlag 1979.

PAPER 14 – NUMBER THEORY I

Unit 1: Arithmetical functions and Dirichlet multiplication.

Chapter 2: Sections 2.2 to 2.19

Unit 2: Averages of arithmetical function.

Chapter 3: Sections 3.1 to 3.12.

Unit 3: Congruences, Finite Abelian Groups and their characters.

Chapter 5: Sections 5.1 to 5.9 (Omit 5.10 and 5.11)

Chapter 6: Sections 6.1 to 6.4.

Unit 4: Finite Abelian Groups and Dirichlet Theorem on primes in arithmetic progression.

Chapter 6: Sections 6.5 to 6.10

Chapter 7: All sections except 7.9.

Unit 5: Quadratic residues and quadratic reciprocity law

Chapter 9: Sections 9.1 to 9.9 (Omit 9.10 and 9.11).

Books for Reference and study:

Content and treatment as in **Introduction to Number Theory – Tom Apostol** – Narosa Publications.

PAPER 15 – ELECTIVE – III (COMMUTATIVE ALGEBRA)

Unit 1: Preliminaries concerning modules – modules over principle ideal rings.

Unit 2: Roots of unity - finite fields – integral element over a ring – integrally closed rings – algebraic elements over fields – algebraic extensions – conjugate elements – conjugate fields – integers in quadratic fields

Unit 3: Norms and traces – discriminant – the terminology of number fields – cyclotomic fields

Unit 4: Noetherian rings and modules – an application concerning integral elements – some preliminaries concerning ideals

Unit 5: Dedikind rings – the norms of an ideal.

Books for Reference and study:

Content and treatment as in **Pierre Samuel, Algebraic theory of numbers,** Houghton Mifflin Company, Boston, 1975.

Books for supplementary reading and reference:

1. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addision Wesley Publishing Company, Mass, 1969.

PAPER 16 – COMPLEX ANALYSIS – II

Unit 1: **Riemann zeta function and normal families -** Product development – Extension of $\varsigma(s)$to the whole plane-The zeros of zeta functions –Equicontiunity -Normality and compactness-Arzela's theorem-Families of analytic functions –The Classical Definition

Chapter 5: Sections 4.1to 4.4

Chapter 5: Sections 5.1 to 5.5

Unit 2: **Riemann mapping theorem -** Statement and Proof-Boundary Behaviour – Use of the Reglection Principle - Conformal mappings of polygons - Behaviour at an angle – Schwarz-Chirstoffel formula – Mappings on a rectangle. Harmonic Functions - Functions with Mean value property – Harnack's principle.

Chapter 6: Sections 1.1 to1.3(omit Section1.4)

Chapter 6: Sections 2.1to2.3(Omit Section 2.4)

Chapter 6: Sections 3.1to 3.2

Unit 3: **Elliptic functions -**Simply periodic functions –Doubly periodic functions

Chapter 7: Sections 1.1to 1.3

Chapter 7: Sections 2.1 to 2.4

Unit 4: W**eiestrass theory -** The weiestrass $\wp$-function-The functions $\zeta(s)$ and $\sigma(s)$ –The differential equation –The modular equation $\lambda(\tau)$- The conformal mapping by $\lambda(\tau)$.

Chapter 7: Sections 3.1 to 3.5

Unit 5: **Analytic Continuation:** The Weierstrass theory – Germs and Sheaves –Sections and Riemann surfaces –Analytic Continuation along Arcs – Homotopic curves –The Monodromy Theorem-Branch points

Chapter 8: Sections 1.1 to 1.7

Books for Reference and study:

Content and treatment as in **Lars f.Ahlfors, Complex Analysis, (3rd Edition),** Mc Graw Hill Book Company; New York, 1979.

Books for supplementary reading and reference:

1. H.A. Prestly, Introduction to Complex Analysis, Clarend Press, Oxford, 1990.
2. J.B. Conway, Functions of one Complex Variable, Springer – Verlag, International Student Edition, Naraso Publishing Co.
3. E. Hille, Analytic Function Theory, (2 Vols), Gonm & Co, 1959.
4. M. Heins, Complex Function Theory, Academic Press, New York, 1968.
5. 5. V. Karunkaran, Complex Analysis, Narosa, 2002.

PAPER 17 – FUNCTIONAL ANALYSIS

Unit 1: Banach Spaces - Definition-some examples –continous linear transformations –the Hahn–Banach theorem - natural embedding of N in N**

Chapter 9: Sections 46 to 49.

Unit 2: Banach spaces and Hilbert Spaces - Open mapping theorem –conjugate of an operator –definition and some simple properties –orthogonal complements-orthonormal sets.

Chapter 9: Sections 50 and 51.

Chapter 10: Sections 52, 53 and 54.

Unit 3: Hilbert Space - conjugate space H*-adjoint of an operator –self-adjoint operator-normal and unitary operators –projections

Chapter 10: Sections 55 to 59.

Unit 4: Preliminaries on Banach algebras - Definition and some examples –regular and singular elements – topological divisors of zero – spectrum –the formula for spectral radius – the radical and semi simplicity.

Chapter 12: Sections 64 to 69.

Unit 5: Structure of Banach Algebras - Gelfand mapping –applications of the formula r(x)= lim ||xn|| 1/n – involutions in Banach algebras – Gelfand Neumark theorem.

Chapter 13: Sections 70 to 73.

Books for Reference and study:

Content and treatment as in **G. F. Simmons – Introduction to Topology and Modern Analysis** - McGraw Hill International Book company, New York 1963.

Books for supplementary reading and reference:

1. W. RUDIN functional analysis, Tata McGraw hill publishing company, New Delhi,1973.
2. G. Bachman and L. Narici, Functional analysis, Academic Press, New York 1966.
3. H. C. Goffman and Pedrick, first course in functional analysis, Prentice Hall of India, New Delhi, 1987.
4. E. Kreyszig, Introductory functional analysis with applications, John Wiley and Sons, New York, 1978.

PAPER 18 – PROBABILITY THEORYAND MATHEMATICAL STATISTICS

Unit 1: Random events and random variables: Random events – Probability axioms-combinatorial formulae – conditional probability – Bayes theorem - Independent events – random variables – Distribution function – join distribution – marginal distribution – conditional distribution – independent random variables – functions of random variables. (Chapter 1: Sec 1.1 to 1.7 / Chapter 2: Sec 2.1 to 2.9)

Unit 2: Parameters of the distribution: Expectation – Moments-The Chebyshev inequality – Absolute moments – Order parameters – Moments of random vectors – Regression of the first and second types. (Chapter 3: Sec 3.1 to 3.8)

Unit 3: Characteristic functions: Properties of characteristic functions – Characteristic functions and moments – semi invariants – characteristic function of the sum of the independent random variables – Determinations of distribution function by the characteristic function – characteristic of multidimensional random vectors – Probability generating functions.
(Chapter 4: Sec 4.1 to 4.7 / Chapter 5: Sec 5.1 to 5.10 (omit sec 5.11))

Unit 4: Limit theorems: Stochastic convergence – Bernoulli law of large numbers – Convergence of sequence of distribution functions – Levy Cramer theorems – de Moivre-Laplace theorem – Poisson theorem –
(Chapter 6: Sec 6.1 to 6.4, 6.6 to 6.9, 6.11 and 6.12 (omit 6.5, 6.10, 6.13 to 6.15))

Unit 5: Sample moments and their functions – notion of a sample – distribution functions - χ^2 distribution – student t-distribution – Fisher's Z-distribution – F-distribution – distribution of sample mean from non-normal populations .(Chapter 9: Sect 9.1 to 9.8)

Books for Reference and study:

Content and treatment as in **M. Fisz, Probability Theory and Mathematical Statistics**, John Wiley and Sons, New York, 1963.

Books for supplementary reading and reference:

1. R. B. Ash, Real analysis and probability, Academic press, New York, 1972.
2. K. L. Chung, A course in Probability, Academic press, New York, 1974.
3. Y. S. Chow and H. Teicher, Probability Theory, Springer Verlag, Berlin, 1988 (2nd edition)
4. R. Durrett, Probability: Theory and Examples, (2nd Edition) Duxbury Press, New York, 1996.
5. V. K. Rohatgi, An introduction to probability theory and Mathematical Statistics, Wiley Eastern Ltd., New Delhi, 1988(3rd print).
6. S. I. Resnick, A Probability Path, Birhauser, Berlin, 1999.
7. B. R. Bhat, Modern Probability Theory (3rd edition), New age international (P) Ltd, New Delhi, 1999.
8. J. P. Romano and A. F. Siegel, Counter Examples in Probability and Statistics, Wadsworth and Brooks/ Cole Advanced Books and Software, California, 1968.

PAPER 19– CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

UNIT-I: The Method of Variations in Problems with Fixed Boundaries.

Chapter 6: sections 1 to 7 (Elsgolts)

UNIT-II: Variational Problems with Moving Boundaries and certain other problems and sufficient conditions for an Extremum.

Chapter 7: sections 1 to 4 (Elsgolts)

Chapter 8: sections 1 to 3 (Elsgolts)

Unit III: Variational problems involving a conditional Extremum.

Chapter 9: sections 1 to 3 (Elsgolts)

Unit IV: Integral Equations with Separable Kernels and Method of successive approximations.

Chapter 1: sections 1.1 to 1.7 (Kanwal) Chapter 2: sections 2.1 to 2.5 (Kanwal)

Chapter 3: sections 3.1 to 3.5 (Kanwal)

Unit V: Classical Fredholm Theory, Symmetric Kernels and Singular Equations.

Chapter 4: sections 4.1 to 4.5 (Kanwal)

Chapter 7: sections 7.1 to 7.6 (Kanwal)

Chapter 8: sections 8.1 to 8.5 (Kanwal)

RecommendedTexts

For Units I, II, III: L. Elsgolts, Differential Equations and the Calculus of variations, Mir Publishers, Moscow, 1973 (2ndEdition)

For Units IV and V: Ram P. Kanwal, Linear Integral Equations, Academic Press, NewYork, 1971

Reference Books

1. I.M.Gelfand and S.V.Fomin, Calculus of Variations, Prentice-Hall Inc. New Jersey, 1963.
2. A.S.Gupta, Calculus of Variations with Applications, Prentice-Hall of India, New Delhi, 1997.
3. M.Krasnov, A.Kiselev and G.Makarenko, Problems and Exercise in Integral Equations, Mir Publishers, Moscow, 1979.
4. S.G Mikhlin, Linear Integral Equations, Hindustan Publishing Corp. Delhi, 1960.
5. L.A.Pars, An Introduction to the Calculus of variations, Heinemann, London, 1965.
6. R.Weinstock, Calculus of Variations with Applications to Physics and Engineering, McGraw-Hill Company Inc. New York, 1952.

Website and e-Learning Source: https://nptel.ac.in/courses/111104025

PAPER 20 – ELECTIVE – IV (REPRESENTATION THEORY OF FINITE GROUPS)

Unit 1: Introduction – Group characters – Representation modules – Applications, ideals and results from Group Theory.

Unit 2: The regular representations – some fundamental lemmas – the principal indecomposable representations – the radical of a ring – semi-simple rings – the Wedderburn structure theorem for semi-simple rings.

Unit 3: Intertwining Numbers – multiplicities of the indecomposable representation – the Generalized Burnside Theorem.

Unit 4: The group algebra – the regular representations of groups – Semisimplicity of group algebra – the centre of the group algebra – the number of inequivalent irreducible representations – relations on the irreducible characters

Unit 5: Algebraic numbers – some results from theory of characters – normal subgroups and character tables (A) the existence of normal subgroups (B) The determinations of all normal subgroups – some classical theorems.

Books for Reference and study:

Content and treatment as in **Martin Buroow, Representation theory of finite groups**, Academic Press.

ELECTIVE SUBJECTS SYLLABUS

1. ADVANCED NUMBER THEORY

UNIT-I: The Eisenstein series and the invariance of g2 and g3 – the discriminant Δ –Klein's modular function $J(\tau)$ – invariance of J and Unimodular transformations – Fourier expansions of $g2(\tau)$ and $g3(\tau)$- Fourier expansion of $\Delta(\tau)$ and $J(\tau)$.

Chapter 1: Sections 1.9 to 1.15

UNIT-II: Mobius transformations – the modular group – the fundamental regions – modular functions – special values of J – modular functions as rational functions of J – mapping properties of J.

Chapter 2: Sections 2.1 to 2.7 (Omit 2.8 and 2.9).

Unit III: Modular forms with multiplicative coefficients – introduction – modular forms of weight k – the weight formula for zeroes of an entire modular form – representation of entire forms in terms of G4 and G6 – the linear space Mk and the subspace Mk,0 – classification of entire forms in terms of their zeroes

Chapter 6: Sections 6.1 to 6.6.

Unit IV: The Hecke operators Tn – transformations of order n – behaviour of Tnf under the modular group – multiplicative property of Hecke operators – eigen functions of Hecke operators – properties of simultaneous eigenforms – examples of normalizes simultaneous eigenforms – estimates for the Fourier coefficients of entire forms – modular forms and Dirichlet series.

Chapter 6: Sections 6.7 to 6.13 – 6.15 and 6.16. (omit 6.14).

Unit V: Kronecker's theorem with applications – approximating real numbers by rational numbers – Dirichlet's approximation theorem – Liouville's approximation theorem – Kronecker's approximation theorem to the one dimensional case only – The Dedekind eta function –Introduction-Siegels's proof of theorem 3.1– infinite product representation for Δ.

Chapter 7: Sections 7.1 to 7.4

Chapter 3: Sections 3.1 to 3.3.

Books for Reference and study:

1. Tom Apostol, Modular Functions and Dirichlet Series in Number Theory, Springer Verlag.
2. Modular Forms, Toshitsune Miyake, Springer Berlin, Heidelberg

2. FLUID DYNAMICS

Unit I: Introductory notions – Velocity – Stream lines, and Path of the particle – Stream tubes and Filaments – Fluid body – Density - Pressure, Differentiation with respect to time – Equation of continuity - Boundary conditions (Kinematical and Physical) - Rate of change of linear momentum – Equation of motion of an inviscid Fluid.

Chapter I: Sections 1.0 – 1.3

Chapter III: Sections 3.10, 3.20, 3.30, 3.31, 3.40, 3.41

Unit II: Euler's Momentum theorem - Conservative forces – Steady motion (Bernoulli's equation) – The Energy equation – Rate of Change of Circulation (Kelvin's theorem) – Vortex motion- Permanence of Vorticity (Helmholtz Equation).

Chapter III: Sections 3.42, 3.43, 3.45, 3.50, 3.51, 3.52, 3.53

Unit III: Two dimensional motions – Two dimensional functions – complex potential basic singularities – source vertex – doublet circle theorem – flow past a circular cylinder with circulation – conformal transformation – Blasius theorem – Lift force.

Chapter III: Sections 3.1 – 3.7.5 (omit 3.4 , 3.5 , 3.5.3 and 3.6)

Unit IV: Viscous flows – Navier stokes equations – Vorticity and Circulation in a viscous fluid – steady flow through an arbitrary cylinder under pressure – steady couette flow between cylinders in relative motion – steady flow between parallel planes. Chapter V: Sections 5.2 – 5.3.3

Unit V: Laminar boundary layer in incompressible flow boundary layer concept – Boundary layer equations – Displacement thickness – Momentum thickness – Kinetic energy thickness – Integral equation of boundary layer – Flow parallel to semi-infinite flat plate - Blasius equation and it's solution.

Chapter VI: Sections 6.2.1, 6.2.3, 6.2.4, 6.3.1

Recommended Text

1. L. M. Milne Thomson, Theoretical Hydrodynamics, Macmillan Company, 5th Edition, 1968. For Units I and II.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics – Vol I, D Van Nostrand Company Ltd; London, 1968. For Units III, IV and V.

Reference Books

1. F. Chorlton, Text book of Fluid dynamics, CBS Publishers 2004.
2. A. J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics, Springer Verlag, New York, 1993.
3. G.K.Batchelor,An Introduction toFluid Dynamics, Cambridge Mathematical Library, 2000.
4. M. D. Raisinghania, Fluid Dynamics, S. Chand, 5th Edition, 2018.

3. STOCHASTIC PROCESSES

Unit I: Elements of Stochastic Processes Review of Basic Terminology and Properties of Random Variables and Distribution Functions; Two Simple Examples of Stochastic Processes; Classification of General Stochastic Processes; Defining a Stochastic Process; Elementary Problems.

Unit II: Discrete-time Markov Chains Definitions; Examples of Markov Chains; Transition Probability Matrices of a Markov Chain; classification of States of a Markov Chain; Recurrence; Examples of Recurrent Markov Chains; More on Recurrence; Elementary Problems.

Unit III: The Basic Limit Theorem of Markov Chains and Applications Discrete Renewal Equation; Proof of Basic Limit Theorem; Absorption Probabilities; Criteria for Recurrence; A Queueing Example; Another Queueing Model; Random Walk; Elementary Problems.

Unit IV: Continuous-time Markov Chains General Pure Birth Processes and Poisson Processes; More about Poisson Processes; A Counter Model; Birth and Death Processes; Differential Equations of Birth and Death Processes; Examples of Birth and Death Processes; Birth and Death Processes with Absorbing States; Finite State Continuous Time Markov Chains; Elementary Problems.

Unit V: Renewal Processes Definition of a Renewal Process and Related Concepts; Some Examples of Renewal Processes; More on Some Special Renewal Processes; Renewal Equations and the Elementary Renewal Theorem; The Renewal Theorem; Applications of the Renewal Theorem; Superposition of Renewal Processes; Elementary Problems.

TEXT BOOK:

S. Karlin & H. M. Taylor: A First Course in Stochastic Processes. Second Edition. Academic Press, New York, 1975.

REFERENCE BOOKS:

1. Chung .K. L., Elementary probability theory with stochastic processes. Undergraduate Texts in Mathematics. Springer, New York, 1974.
2. Edward. P. C. Kao, An Introduction to Stochastic Processes. Dover 2019, Duxbury Press, 1997.
3. Mehdi .J, Stochastic Processes. Third Edition. New Academic Science, 2009.
4. Srinivasan S.K. and K.M.Mehata, Stochastic Processes, Tata McGraw Hill, 1976.
5. Cox D.R and Miller. H.D, Theory of Stochastic Processes, Chapman and Hall, London, Third Edition,1983.
6. Sheldon M Ross , Stochastic Processes, second Edition, Wiley.1995.
7. Oliver C.Ibe ,Fundamentals of Applied Probability and Random Processes, University of Massachusetts Lowell, MA, Academic Press,2007.
8. Bhat.N., Elements of Applied Stochastic Processes, John Wiley, 2nd Edition, 1984.